Level – 2	Daily Tutorial Sheet - 8

101. $\displaystyle\int \frac{x-1\ dx}{(x+1)^{3/4}(x+2)^{5/4}} = \int \frac{x-1}{\left[\frac{x+1}{x+2}\right]^{3/4}(x+2)^2}dx$

$\displaystyle\frac{x+1}{x+2} = t^4 \Rightarrow \frac{(x+2)-(x+1)}{(x+2)^2}dx = 4t^3dt$ and $\displaystyle x = \frac{2t^4-1}{1-t^4}$

$$\int \frac{\left(\frac{3t^4-2}{1-t^4}\right)}{t^3}(4t^3dt) = \int \frac{3(t^4-1)+4}{1-t^4}dt = -3t - \frac{3}{2}\int\left[\frac{1}{t^2-1} - \frac{1}{t^2+1}\right]dt$$

102. $\displaystyle I = \int \frac{\sin(x-\alpha)dx}{\sqrt{\sin(x+\alpha)\sin(x-\alpha)}} = \int \frac{\sin x\cos\alpha\, dx}{\sqrt{\cos^2\alpha - \cos^2 x}} - \int \frac{\cos x \sin\alpha}{\sqrt{\sin^2 x - \sin^2\alpha}}dx$

(Substitute $t = \cos x$) (Substitute $z = \sin x$)

103.(C) $\displaystyle\int x^{13/2}\cdot(1+x^{5/2})^{1/2}dx = \int x^5.x^{3/2}.(1+x^{5/2})^{1/2}dx = \int x^5.\frac{4}{5}z.z\,dz$

$\left[\text{Putting } 1+x^{5/2} = z^2, \frac{5}{2}x^{3/2}dx = 2z\,dz \text{ i.e. } x^{3/2}dx = \frac{4}{5}z\,dz\right]$

$$= \frac{4}{5}\int z^2.(z^2-1)^2dz = \frac{4}{5}\int z^2(z^4-2z^2+1)dz = \frac{4}{5}\left[\frac{z^7}{7} - \frac{2z^5}{5} + \frac{z^3}{3}\right] + C$$

$$= \frac{4}{35}(1+x^{5/2})^{7/2} - \frac{8}{25}(1+x^{5/2})^{5/2} + \frac{4}{15}(1+x^{5/2})^{3/2} + C \qquad \therefore \qquad A = \frac{4}{35}, B = \frac{-8}{25} \text{ and } C = \frac{4}{15}.$$

104.(C) $\displaystyle\int \frac{\sqrt{x^2+1}\left[\log(x^2+1) - 2\log(x)\right]}{x^4}dx = \int\left(\int\sqrt{1+\frac{1}{x^2}}\right).\frac{1}{x^3}.\log\left(1+\frac{1}{x^2}\right)dx$

$$= \frac{-1}{2}\int\sqrt{z}\log z\,dz \quad \left[\text{Putting } 1+\frac{1}{x^2} = z \Rightarrow \frac{1}{x^3}dx = \frac{-1}{2}dz\right] = \frac{-1}{2}\left[\log z.\frac{2z^{3/2}}{3} - \int\frac{1}{z}.\frac{2}{3}z^{3/2}dz\right]$$

$$= \frac{-1}{3}z^{3/2}\log z + \frac{2}{9}z^{3/2} + C = \frac{z^{3/2}}{3}\left(\frac{2}{3} - \log z\right) + C = \frac{(x^2+1)^{3/2}}{3x^3}\left[\frac{2}{3} - \log\left(\frac{x^2+1}{x^2}\right)\right] + C$$

105.(C) Put $\displaystyle\frac{1}{x\sin x + \cos x} = z$

$$\Rightarrow \quad \frac{-1}{(x\sin x + \cos x)^2}(\sin x + x\cos x - \sin x)dx = dz$$

$$\Rightarrow \quad \frac{x\cos x}{(x\sin x+\cos x)^2}dx = -dz \qquad \therefore \qquad \int\frac{x^2}{(x\sin x + \cos x)^2}dx$$

$$= \int\frac{x}{\cos x}.\frac{x\cos x}{(x\sin x+\cos x)^2}dx = \frac{x}{\cos x}.(-z) - \int(-z)\left(\frac{\cos x.1 + x\sin x}{\cos^2 x}\right)dx$$

$$= \frac{-x}{\cos x(x\sin x+\cos x)} + \int \sec^2 x dx = \frac{-x}{\cos x(x\sin x+\cos x)} + \tan x + C \quad \Rightarrow \quad f(x) = \frac{-x}{\cos x}.$$

106.(D) $I = \int e^x \left(\frac{x}{\sqrt{1+x^2}} + \frac{1}{(1+x^2)^{3/2}} \right) dx$

$$\text{Let } f(x) = \frac{x}{\sqrt{1+x^2}} \Rightarrow f'(x) = \frac{\sqrt{1+x^2} - \frac{x^2}{\sqrt{1+x^2}}}{(1+x^2)} \Rightarrow f'(x) = \frac{1+x^2-x^2}{(1+x^2)\sqrt{1+x^2}} = \frac{1}{(1+x^2)^{3/2}}$$

$$\Rightarrow I = e^x f(x) + c = \frac{e^x x}{\sqrt{1+x^2}} + c.$$

107.(B) Put $\frac{2-x}{2+x} = z^3 \Rightarrow x = \frac{2-2z^3}{1+z^3} \Rightarrow dx = \frac{-12z^2 dz}{(1+z^3)^2}$

$$\therefore \int \frac{2}{(2-x)^2}\sqrt[3]{\frac{2-x}{2+x}}dx = \int \frac{2}{\left(2-\frac{2-2z^3}{1+z^3}\right)}.z.\frac{(-12z^2)}{(1+z^3)^2}dz = \frac{-3}{2}\int \frac{dz}{z^3} = \frac{3}{4z^2} + C = \frac{3}{4}\left(\frac{2+x}{2-x}\right)^{2/3} + C.$$

108.(B) Put $1+x^{1/4} = z^3 \Rightarrow dx = 12z^2x^{3/4}dz \quad \therefore \quad \int \frac{\sqrt[3]{1+\sqrt[4]{x}}}{\sqrt{x}}dx = \int \frac{z}{x^{1/2}}.(12z^2x^{3/4})dz$

$$= 12\int z^3.x^{1/4}dz = 12\int z^3(z^3-1)dz = 12\left(\frac{z^7}{7} - \frac{z^4}{4}\right) + C = 12\left(\frac{(1+x^{1/4})^{7/3}}{7} - \frac{(1+x^{1/4})^{4/3}}{4}\right) + C$$

109.(A) Put $1+x^{4/3} = z^7 \Rightarrow x^{1/3}dx = \frac{21}{4}z^6 dz$

$$\therefore \int \sqrt[3]{x}\sqrt[7]{1+\sqrt[3]{x^4}}\,dx = \int z\left(\frac{21}{4}z^6 dz\right) = \frac{21}{4}.\frac{z^8}{8} + C \quad \Rightarrow \quad \frac{21}{32}\left(1+\sqrt[3]{x^4}\right)^{8/7} + C.$$

110.(C) $\int \frac{(x-x^5)^{1/5}}{x^6}dx = \int \frac{\left(\frac{1}{x^4}-1\right)^{1/5}}{x^5}dx = \frac{-1}{4}\int t^{1/5}dt \quad \left[\text{Putting } \frac{1}{x^4} - 1 = t \Rightarrow \frac{-1}{4}dt\right]$

$$= \frac{-5}{24}t^{6/5} + C = \frac{-5}{24}\left(\frac{1}{x^4}-1\right)^{6/5} + C.$$